Α Π Ο Λ Υ Τ Η Τ Ι Μ Η

Α. Ορισμός: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Β. Ιδιότητες

1. $|x| \geq 0, \quad \forall x \in IR$

2. $|x| = |-x|, \quad \forall x \in IR$

3. $-|x| \leq x \leq |x|, \quad \forall x \in IR$

4. $|x^{2\nu}| = |x|^{2\nu} = x^{2\nu}, \quad x \in IR, \quad \nu \in \mathrm{N}$

5. $|x| = \alpha \Leftrightarrow \begin{cases} x = \pm\alpha, \alpha\nu\ \alpha \geq 0 \\ \text{Αδύνατη}, \alpha\nu\ \alpha < 0 \end{cases}$

6. $|x| \leq \alpha \Leftrightarrow \begin{cases} -\alpha \leq x \leq \alpha, \alpha\nu\ \alpha > 0 \\ 0, \alpha\nu\ \alpha = 0 \\ \text{Αδύνατη}, \alpha\nu\ \alpha < 0 \end{cases}$

7. $|x| > \alpha \Leftrightarrow \begin{cases} x < -\alpha\ \eta\ x > \alpha, \alpha\nu\ \alpha > 0 \\ x \in IR^{*}, \alpha\nu\ \alpha = 0 \\ x \in IR, \alpha\nu\ \alpha < 0 \end{cases}$

8. $|\alpha + \beta| \leq |\alpha| + |\beta|, \quad (\tau\acute{o} = \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \acute{o}\tau\alpha\nu\ \alpha\beta \geq 0)$

9. $|\alpha - \beta| \leq |\alpha| + |\beta|, \quad (\tau\acute{o} = \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \acute{o}\tau\alpha\nu\ \alpha\beta \leq 0)$

10. $|\alpha_1 + \alpha_2 + \cdots + \alpha_\nu| \leq |\alpha_1| + |\alpha_2| + \cdots + |\alpha_\nu|, \quad \nu \in \mathrm{IN}^{*}$

11. $|\alpha \cdot \beta| = |\alpha| \cdot |\beta|$

12. $\left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|}, \quad \beta \neq 0$

13. $|\alpha_1 \cdot \alpha_2 \cdots \alpha_\nu| = |\alpha_1| \cdot |\alpha_2| \cdots |\alpha_\nu|, \quad \nu \in \mathrm{IN}^{*}$

14. $\big||\alpha| - |\beta|\big| \leq |\alpha \pm \beta| \leq |\alpha| + |\beta|$